Stability of parallel queueing systems with coupled service rates

S. Borst*†, M. Jonckheere*, L. Leskelä*

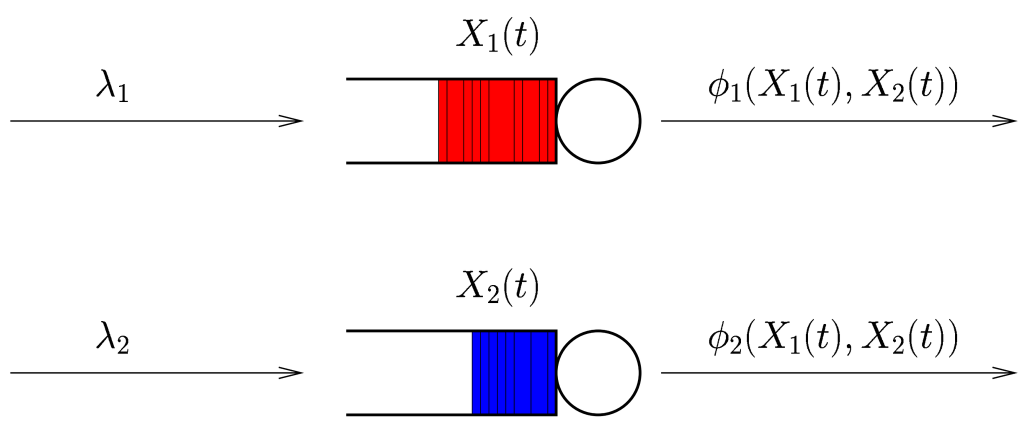
*Centrum voor Wiskunde en Informatica
and
†Technische Universiteit Eindhoven

`sem@win.tue.nl`, `matthieu.jonckheere@cwi.nl`, `lasse.leskela@iki.fi`

Introduction

We study a parallel system of queues fed by independent arrival streams, where the service rate ϕ_i for queue i is a function of the number of customers in all of the queues. These models are natural for:

- *Wireless networks*, where the available transmission rate in a particular cell depends on the number of ongoing data transfers in the neighboring cells.
- *Manufacturing systems*, where a server is capable to process other queues when its own buffer is empty.



Capacity planning problem. *How big load can the system support while remaining stochastically stable?*

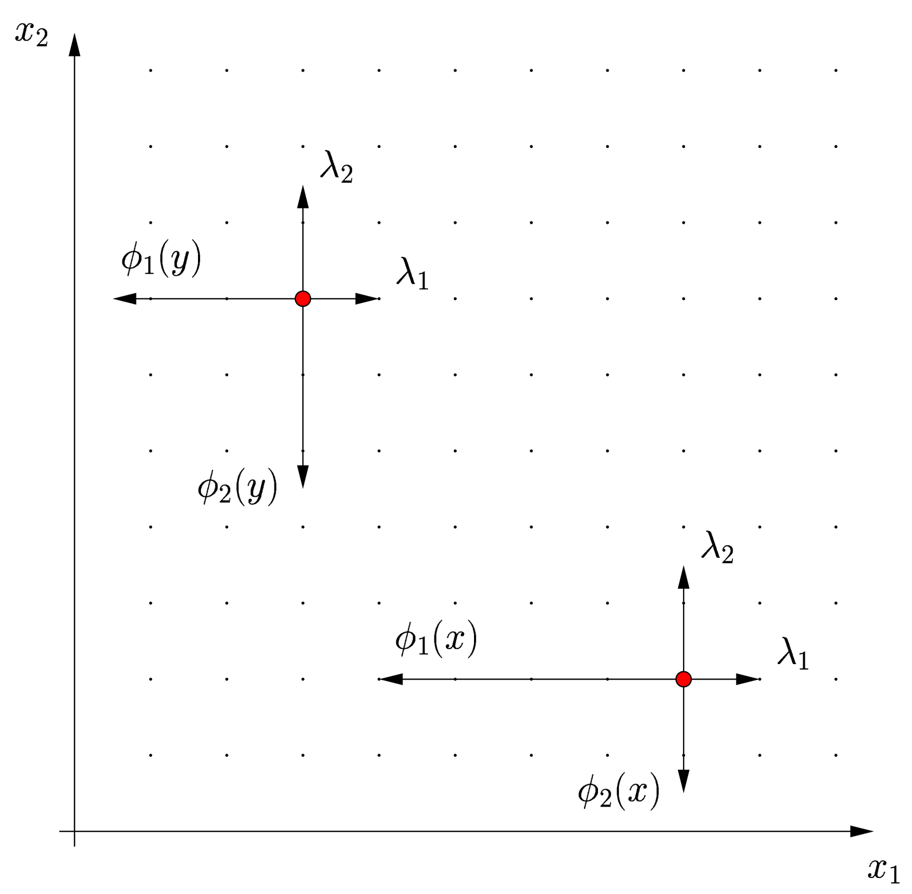
In mathematical terms, determine the set of input rates $(\lambda_1, \ldots, \lambda_N)$, for which the queue length process $X = (X_1, \ldots, X_N)$ satisfies

$$\lim_{r\to\infty} \sup_{t\geq 0} \mathrm{P}(|X(t)| > r) = 0.$$

Monotone systems

A Markov process X is *monotone*, if for all increasing functions f, the map $x \mapsto \mathrm{E}\{f(X(t)) \,|\, X(0) = x\}$ is increasing.

Proposition. A parallel queueing system with state-dependent service rates $\phi_1, \ldots, \phi_N$ is monotone if and only if $x_j \mapsto \phi_i(x_1, \ldots, x_N)$ is decreasing for all i and all $j \neq i$.



Main results

For arbitrary parallel queueing systems:

- Conditions for verifying the stability/unstability of queue i.

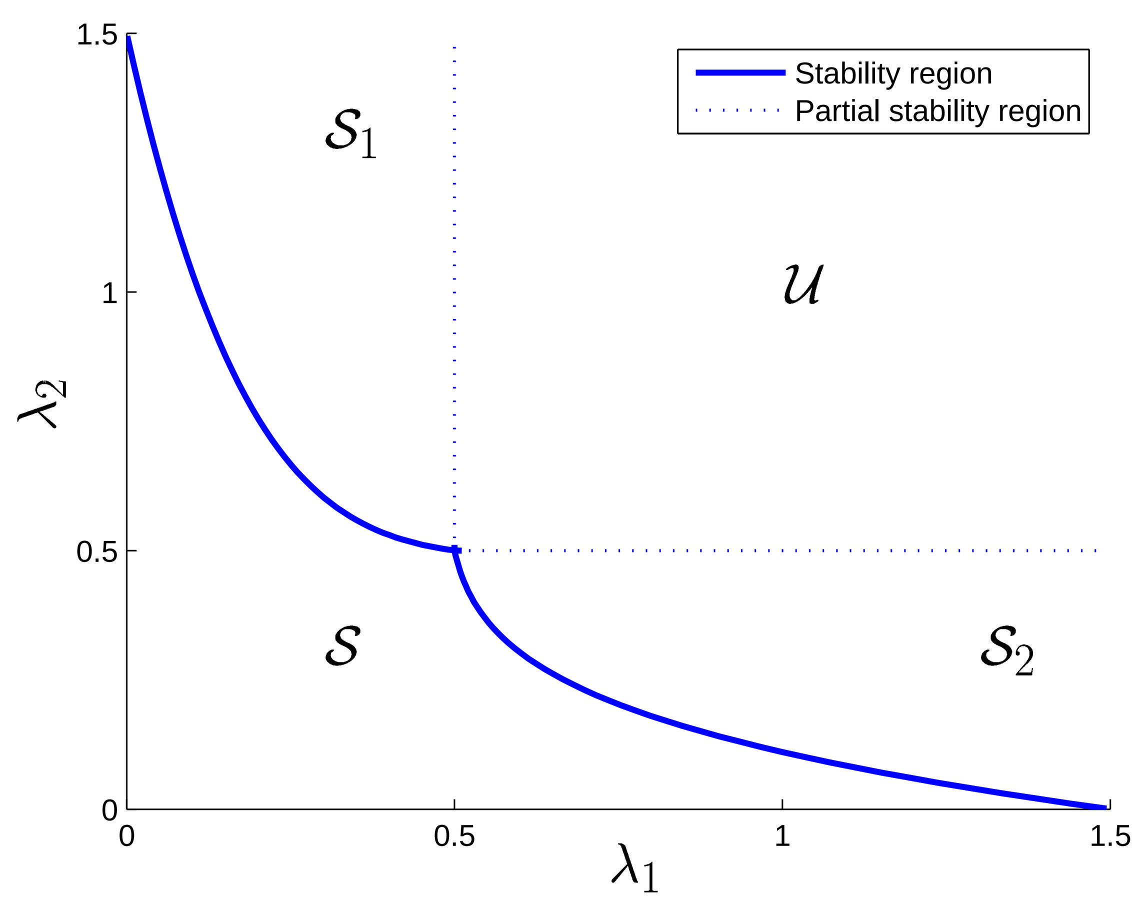
For monotone systems where ϕ_i has uniform limits in infinity:

- Sharp characterization of the stability region.

As an example, consider two neighboring wireless base stations, where the number of ongoing file transfers is modeled as a parallel queueing system with

$$\phi_i(x) = \frac{\min(3, \log(1 + x_i))}{6 - 4e^{-2x_j}}, \quad j \neq i,$$

where the numerator represents multiuser diversity gain and the denominator models intercell interference. The figure below describes the stability regions of the system: both queues are stable for $(\lambda_1, \lambda_2) \in \mathcal{S}$; only queue i is stable for $(\lambda_1, \lambda_2) \in \mathcal{S}_i$; and both queues are unstable for $(\lambda_1, \lambda_2) \in \mathcal{U}$.



References

[1] Borst, S., Jonckheere, M., and Leskelä, L. (2006). Stability of parallel queueing systems with coupled service rates. Preprint: CWI Report PNA-E0613.

Acknowledgements

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