Teknillinen korkeakoulu Alestalo
Matematiikka

Mat-1.1620 Mathematics II

3rd partial exam 7.5.2008, 16–19.

You may use a calculator but no "Formula books".

1. Calculate the line integral
$$\int_C \mathbf{F}\cdot\mathbf{dr} = \int_C xy\,dx - (x+y)\,dy,$$
where C is the line segment from $(0,0)$ to $(3,2)$.
Note: The above notation means that $\mathbf{F}(x,y) = xy\mathbf{i} - (x+y)\mathbf{j}$.

2. Calculate the z-coordinate
$$\bar{z} = \frac{1}{A}\iint_P z\,dS.$$
of the centroid of the hemisphere of radius R:
$$P = \{(x,y,z) \mid x^2+y^2+z^2 = R^2,\ z \geq 0\}.$$
Hint: The area $A = 2\pi R^2$ and the formula $z = R\cos\varphi$ on the sphere can be taken for granted.

3. According to Green's Theorem in the plane we have
$$\oint_{\partial R} F_1\,dx + F_2\,dy = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\,dA,$$
where ∂R is the boundary curve of the plane region R. Using the formula,
a) explain, why the area of R is given by the line integral
$$A = \oint_{\partial R} x\,dy.$$
b) find line integrals that give the plane integrals
$$\iint_R x\,dA \quad \text{and} \quad \iint_R y\,dA$$
needed in the calculation of the centroid of R. (Note: the results are not unique)
c) Explain the values of both sides of the Theorem if the vector field $\mathbf{F}$ has a potential; i.e. $\mathbf{F} = \nabla\varphi$.

4. a) Find a solution of the differential equation $y' = y/(1+2x)$ satisfying the condition $y(0) = 3$.
b) Find the general solution of $y'' - y' - 6y = 12e^{-t}$.
Hint: The particular solution is of the form Ae^{-t}.