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Matematiikka

Mat-1.1620 Mathematics II

2nd partial exam 1.4.2008, 16–19.

You may use a calculator but no "Formula books".

1. Let $f(x, y) = \cos(x + y) + \sin(x - y)$.
   a) Show that the critical points of f are obtained from the equations
   $$\begin{cases} \sin(x+y) = 0 \\ \cos(x-y) = 0. \end{cases}$$
   b) Classify the critical point $(\pi/4, 3\pi/4)$ as a local max/min/saddle.

2. Let $x, y, z > 0$ and $xyz = 1$. Show that
   $$x + y + z \geq 3.$$
   Hint: Find the minimum of $x + y + z$ under the condition $xyz = 1$.

3. Sketch the domain of integration for the iterated integral
   $$\int_0^{\pi/2} dy \int_y^{\pi/2} \frac{\sin x}{x}\, dx$$
   and evaluate it by changing the order of integration.
   Note: You may regard $\frac{\sin x}{x}$ as a continuous function at $x = 0$, so this is not an improper integral.

4. a) Using polar coordinates, calculate the y-coordinate of the centroid of the upper half disk $D = \{(x, y) \mid x^2 + y^2 \leq R^2,\ y \geq 0\}$; that is,
   $$\bar{y} = \frac{1}{A} \iint_D y\, dA.$$
   b) The temperature $T = T(\rho)$ of a ball B of radius R decreases linearly (with respect to ρ) from the value 100 to 0; i.e. $T(\rho) = 100(1 - \rho/R)$ for $0 \leq \rho \leq R$. Calculate the mean temperature
   $$\frac{1}{V} \iiint_B T\, dV.$$
   of the ball.