Linear algebraic aspects of exponential integrators

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We condsider the time integration of large stiff systems of ordinary differential equations, which result from semidiscretization of partial differential equations and which are of the form $u'(t) = Au(t) + g(u(t))$. We introduce so called exponential Taylor methods, which can be obtained by Taylor series expansion of the nonlinear part at each numerical approximation. We also shortly discuss the error analysis in an abstract framework, where the error bounds are independent of the norm of the linear operator.

When implementing exponential integrators, the size of the operator plays however a crucial role, as one needs to evaluate the action of a matrix exponential (and closely related functions) on a vector. We discuss the use of Krylov subspace methods for these problems. We introduce a so called moment-matching Arnoldi iteration for computing series of the form $\exp(hA)u_n + \sum_{k=1}^{p} h^k \varphi_k(hA) w_k$, where the matrix functions φ_k are related to the exponential function, and which appears when implementing exponential integratos. Using Cauchy's integral formula we give a representation for the error of the moment-matching approximation and derive a priori error bounds which describe well the convergence behaviour of the algorithm. In addition an efficient a posteriori estimate is derived.