Computational Methods in Inverse Problems, Mat–1.3626

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Consider the deconvolution problem with Gaussian additive noise

$$B = AX + E, \quad E \sim \mathcal{N}(0, \sigma^2 I),$$

where

$$a_{jk} = h(s_j - t_k), \quad 1 \leq j, k \leq n, \quad h(t) = \frac{1}{\sqrt{2\pi\omega^2}} \exp\left(-\frac{1}{2\omega^2} t^2\right),$$

and the sapling points are $s_j = j/n$, $t_k = k/n$. Assume that we use a *white noise prior*,

$$\pi_{\text{prior}}(x \mid \gamma) = \left(\frac{1}{2\pi\gamma}\right) \exp\left(-\frac{1}{2\gamma}\|x\|^2\right),$$

where the variance γ is not known.

1. Assuming the flat hyperprior (i.e., only positivity of γ is assumed), write a sequential iterative algorithmfor computing the MAP estimate, updating alternatingly x and γ.
2. Interpret the MAP estimate as Tikhonov regularization and estimate the parameter γ from the Morozov discrepancy principle.
3. Compare the performance of the above algorithms with different input signals and different noise levels.

   Which one performs better? Is the performance different with different noise levels or with different inputs? Does the prior variance parameters correspond to the amplitude of the true input signal?