Computational Methods in Inverse Problems, Mat–1.3626

Fall 2007 Erkki Somersalo/Knarik Tunyan

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1. Implement your own Conjugate Gradient solver based on the algorithm described at the end of the CG lecture notes. Test the program with a simple $n \times n$ symmetric positive definite matrix to see that it converges to the exact solution in n iteration steps.

2. Go to the lecture notes on TSVD and implement for yourself the 60×60 convolution matrix with the Gaussian kernel. Calculate the data using the boxcar function used in the notes, and add noise (define the noise level as you like.) Now you have the model

$$b = Ax_* + e.$$

   Apply your CG algorithm to the linear problem $Ax = b$ to approximate iteratively the solution. Let x_k denote the kth iteration. Follow the evolution of the error and residual by plotting

$$\|e_k\| = \|x_k - x_*\|, \quad \|r_k\| = \|Ax_k - b\|.$$

   You should observe a *semi-convergence*: first the error norm goes down, then it starts to increase again.

3. Select the optimal number of iterations by the discrepancy principle, i.e., stop the iterations when the norm of the residual is of the order of the norm of the error. Plot the corresponding estimates x_k near the optimal stopping value. For comparison, plot also the L-curve. Does the L-curve criterion give a similar value for the optimal stopping value?