Hanna Katriina Pikkarainen: *A Mathematical Model for Electrical Impedance Process Tomography*; Helsinki University of Technology Institute of Mathematics Research Reports A486 (2005).

Abstract: We consider the process tomography problem of the following kind: based on electromagnetic measurements on the surface of a pipe, describe the concentration distribution of a given substance in a fluid moving in the pipeline. We view the problem as a state estimation problem. The concentration distribution is treated as a stochastic process satisfying a stochastic differential equation. This is referred to as the state evolution equation. The measurements are described in terms of an observation equation containing the measurement noise. The time evolution is modelled by a stochastic convection-diffusion equation. The measurement situation is represented by the most realistic model for electrical impedance tomography, the complete electrode model. In this thesis, we give the mathematical formulation of the state evolution and observation equations and then we derive the discrete infinite dimensional state estimation system. Since our motive is to monitor the flow in the pipeline in real time, we are dealing with a filtering problem in which the estimator is based on the current history of the measurement process. For computational reasons we present a discretized state estimation system where the discretization error is taken into account. The discretized filtering problem is solved by the Bayesian filtering method.

AMS subject classifications: 62M20 (Primary); 93E10, 60H15, 35J25 (Secondary)

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quantities of primary interest, we are dealing with a *filtering problem* in which the estimator is based on the current history of the measurement process.

Often in state estimation approach the time variable is assumed to be discrete and the space variable to be finite dimensional. This is convenient from the practical point of view. Observations are usually done at discrete time instants and the computation requires space discretization. Hence discrete state evolution and observation equations are needed. They may be derived from the continuous ones, especially if the state evolution and observation equations are linear. In many applications, it is assumed that the discretized version of the discrete infinite dimensional state estimation problem represents the reality. Nevertheless, discretization causes always an error, which should be included into the state estimation system. If we analyse the continuous infinite dimensional state evolution and observation equations, we may be able to present the distribution of the discretization error. The discretized filtering problem can be solved by the *Bayesian filtering method*. The discretized state evolution equation is used to find the prior distribution and the likelihood function is given by the discretized observation equation. The solution to the filtering problem is the posterior distribution given by the Bayes formula. As an example of nonstationary inverse problem we examine the electrical impedance process tomography problem.

1.1 Electrical Impedance Process Tomography

In this thesis we consider the process tomography problem of imaging the concentration distribution of a given substance in a fluid moving in a pipeline based on electromagnetic measurements on the surface of the pipe. In electrical impedance tomography (EIT) electric currents are applied to electrodes on the surface of an object and the resulting voltages are measured using the same electrodes (Figure 1.1). The conductivity distribution inside the object is reconstructed based on the voltage measurements. The relation between the conductivity and concentration depends on the process and is usually non-linear. At least for strong electrolytes and multiphase mixtures such relations are studied and discussed in the literature [7, 12]. In traditional EIT it is assumed that the object remains stationary during the measurement process. A complete set of measurements, also called a *frame*, consists of all possible linearly independent injected current patterns and the corresponding set of voltage measurements. In process tomography we cannot in general assume that the target remains unaltered during a full set of measurements. Thus conventional reconstruction methods [4, 5, 6, 46, 47, 49] cannot be used. The time evolution needs to be modeled properly. We view the problem as a state estimation problem. The concentration distribution is treated as a stochastic process that satisfies a stochastic differential equation referred to as the state evolution equation. The measurements are described in terms of an observation equation containing the measurement noise. Our goal is to have a real-time monitoring for the flow in a pipeline. For that reason the computational time has to be minimized. Therefore, we use a simple model, the convection–diffusion equation, for the flow. It allows numerical implementation using FEM techniques. Since we cannot be sure that other features such as turbulence of the flow do not appear, we use stochastic modelling. The measurement situation is represented by the most realistic model for EIT, the *complete electrode model*. The measurements are done in a part of the boundary