Mat-1.3379 Dynaamisten systeemien erikoiskurssi **Eirola/Hannukainen**
Harjoitus 8, 30.3.2006 12-14 Y339B

Harjoitus 8 on tietokoneharjoitus. Tehtäviä tehdään yhdessä assistentin kanssa tietokoneluokassa ja joistain tehtävistä palautetaan lyhyt selostus 6.4. harjoituksiin mennesä.

1. Show, that for every $\boldsymbol{A} \in \mathbb{R}^{n \times n}, \sigma(A) \cap i\mathbb{R} = \phi$, there exists a matrix $\boldsymbol{V}$ such, that
$$\boldsymbol{A} = \boldsymbol{V} \text{diag}\,(\boldsymbol{T}_{+}, \boldsymbol{T}_{-})\,\boldsymbol{V}^{-1},$$
where $\boldsymbol{T}_{-}$ and $\boldsymbol{T}_{+}$ are upper triangular matrixes with eigenvalues with positive/negative real parts. Test your result in Matlab for a set of random matrices. (Hint : use Matlab-function *ordschur* to reoder eigenvalues).

2. Let $f : \mathbb{R}^d \to \mathbb{R}$, $f \in C^{\infty}$. Try to find derivatives of f numerically at point u. Use similar strategy as in Problem 3. Expand f as
$$f(x) = \sum_{j=1}^{k} \gamma_j [x]^j + O(|x|^{k+1}),$$
evaluate f at points $x_1, x_2, \ldots, x_n$, and use Least-Squares method to solve coefficient's γ_j.

   Return (by 6.4) a short eplanation out of the method. Test the method for functions $sin(x)$, $sin(x_1 x_2)$, $sin(x_1 x_2 x_3)$. Vary the number of evaluation points and polynomial degree. Try to use complex evaluation points.

3. (Problem 3.3 p.46) The following is known as the McMillan map :
$$\boldsymbol{\phi}([\begin{smallmatrix} x \\ y \end{smallmatrix}]) \;=\; \begin{bmatrix} y \\ -x + 2y\,(\frac{\mu}{1+y^2} + \epsilon) \end{bmatrix},$$
where $\mu = 2$ and $\epsilon = 0.05$. The origin is a fixed point and it is a saddle. Find the approximations of its stable and unstable manifolds by using the approach presented in *T.Eirola, J.Pfaler: Numerical Taylor Expansions For Invatiant Manifolds*.