Anisotropic Besov Spaces in the Wiener Space

Juha Ylinen (with S. Geiss)
University of Jyväskylä

Abstract:
We introduce the concept of Anisotropic Besov Spaces in the Wiener Space. As an application we deduce L_p-upper bounds of some BSDEs.

Probabilistic approach to solution of the Cauchy problem for quasilinear parabolic equations

Nemchenko E.
St.Petersburg State University for Architecture and Civil Engineering,

We consider two probabilistic representations of a solution to the Cauchy problem for a nonlinear parabolic equation and apply them to construct efefctive algorithms of its numerical solution.

Let $L^u(x)u = \frac{1}{2}A(x,u)\nabla^2 uA(x,u) + \langle a(x,u), \nabla u\rangle$ and $w(t) \in R^d$ be a Wiener process. Given $a(x,u) \in R^d, A(x,u) \in R^{d\times d}, g(x,u) \in R^1, x \in R^d, u \in R^1$ we consider a stochastic process $\xi(t)$ satisfying the system

$$d\xi(t) = a(\xi(t), u(t,\xi(t)))dt + A(\xi(t), u(t,\xi(t)))dw(t), \quad \xi(s) = x,$$

$$u(s,x) = E_{s,x}[u_0(\xi(T))] + \int_s^T g(\xi(t), u(t,\xi(t)))dt].$$

Then under some additional assumptions $u(s,x)$ is a classical solution of the Cauchy problem

$$u_t + L^u u + g(x,u) = 0, \quad u(T,x) = u_0(x). \tag{1}$$

Let $a = a(x), A = A(x), g = g(x,u,A\nabla u)$. Consider a BSDE

$$dy(t) = -g(\xi(t), y(t), z(t))dt + z(t)dw(t), \quad y(T) = u_0(\xi(T),$$

where $\xi(t)$ solves SDE

$$d\xi(t) = a(\xi(t))dt + A(\xi(t))dw(t), \quad \xi(s) = x.$$

Then under some additional assumptions $u(s,x) = y(s)$ solves (1) in the viscosity sense. We apply this representations to develop effective algorithms to find numerical solutions of (1).

Support of grant RFBR Grant 12-01-00457 is gratefully acknowledged.

Probabilistic algorithms to solve boundary value problems for a system of parabolic equations

Romadanova M.M.
St.Petersburg State University for Architecture and Civil Engineering,

Free boundary problem is an important problem that arises in different applications in particular in a description of a visual image in electrography and the process of melting solids, in combustion theory and so on. It appears as well in finacial mathematics when one needs to find an arbitrage free price of an American option. We are interested in the free boundary problem solution for a system with switching regimes.

Let $L^m u^m = \frac{1}{2} A^{m*} \nabla^2 u^m A^m + \langle a^m, u^m \rangle$ and $w(t) \in R^d$ be a Wiener process Consider a stochastic process $\xi(t)$ satisfying SDE

$$d\xi(t) = a(\xi(t), \nu(t))dt + A(\xi(t), \nu(t))dw(t), \xi(s) = x$$

where $\nu(t)$ is a Markov chain independent of $w(t)$ with values in $V = \{1, 2, \ldots, d\}$ having generator $Q = \{q_{lm}\}_{l,m=1}^d$. Let $\nu(s) = m$. Then under some additional assumptions $u^m(s,x) = E_{s,x,m}\left[e^{\int_s^T q_{\nu(t)}(t,\xi(t))dt} \varphi(\xi(T), \nu(T))\right]$ solves the Cauchy problem

$$u_t^m + L^m u^m + \sum_{l=1}^d q_{ml} u_l = 0, \quad u^m(T,x) = \varphi^m(x)$$

and, when $d = 1$, $u^m(s,x) = sup_{\tau \in \mathcal{T}} E_{s,x,m} \left[\exp\left(\int_s^\tau q_\nu(t)(t, \xi(t))dt\right) \varphi(\xi(\tau), \nu(\tau))\right]$, where $\mathcal{T}$ is a set of all stopping times $\tau \in [s,T]$, solves the free boundaray problem

$$\begin{cases} \max\{\frac{\partial u_m}{\partial s} + L^m u^m + \sum_{l=1}^d q_{lm} u_l, \varphi^m - u^m\} = 0 & \text{in } [0,T) \times R, \\ u^m(T,x) = \varphi^m(x) & \text{in } \{T\} \times R \end{cases}$$

in the viscosity sense. We apply this representations to develop effective algorithms to find numerical solutions of above problems

Support of grant RFBR Grant 12-01-00457 is gratefully acknowledged.

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Large and moderate deviations for weighted empirical measures in importance sampling

Pierre Nyquist
KTH

Abstract:

Stochastic simulation has emerged as an important tool for researchers in physics, chemistry, computer science, economics etc. As the computational tasks become increasingly demanding, standard methods such as Monte Carlo can become too costly for practical purposes. Importance sampling, a method invented as a means of reducing the variance in Monte Carlo simulation, is a possible remedy for this.

The output of an importance sampling algorithm can be represented as a weighted empirical measure, where the weights are given by the likelihood ratio between the original distribution and the sampling distribution. We discuss large and moderate deviation results for such weighted empirical measures and the application to efficiency analysis of importance sampling algorithms.

Aleksei Vorotov
St. Petersburg State University, Russia

Occupation time Markov property for countable homogeneous Markov chains.

We consider the occupation time process for homogeneous Markov chain with continuous time and countable state space. It is convenient to consider this chain as a random walk on some graph Γ. The vertex v is called an essential if when we delete it from Γ, the graph will break up into several connectivity components $A_1, \ldots, A_N$ $(N > 1)$. Roughly speaking we threat this components as «past» and «future» and the vertex v as «present». Thus we can talk about the Markov property of the occupation time for the essential vertex v.

It is known that the Markov property is valid for random walk on a tree. We generalize it for an arbitrary graph.

Similarly we can define a pair of vertices $\{v_1, v_2\}$ to be an essential in aggregate and consider the Markov property for them. It turns out that in this case the occupation time process will be Markovian if and only if one of the vertices v_1, v_2 is essential.

There are some other interesting variants of the Markov property of the occupation time and we discuss it too.

On rough asymptotic behaviour of ruin probabilities in a general discrete risk model

Jaakko Lehtomaa
University of Helsinki

Abstract:

We study the rough asymptotic behaviour of a general economic risk model in a discrete setting. Both financial and insurance risks are taken into account. Loss of the year n is modelled as a random variable $B_1 + A_1B_2 + \ldots + A_1 \ldots A_{n-1}B_n$, where A_i corresponds to the financial risk of the year i and B_i represents the insurance risk respectively.

The main result shows that ruin probabilities exhibit power law decay under general assumptions. Our objective is to find the relevant quantities that describe the speed at which the ruin probability vanishes as the amount of initial capital grows. It turns out that these quantities can be expressed as maximal moments, called moment indices, of suitable random variables.

Calibration of Stochastic Volatility Models Using Particle Markov Chain Monte Carlo

Jonas Hallgren
KTH

Keywords: Stochastic volatility; correlation; Bayesian inference; sequential Monte Carlo methods; particle Markov chain Monte Carlo methods.

Abstract:
In this note we use the celebrated particle Markov chain Monte Carlo method proposed by Andrieu, Doucet, and Holenstein (2010) for calibrating a state space stochastic volatility model with possibly non-zero correlation between the driving noise terms of the log-volatility and asset return processes. In this framework, we perform Bayesian parameter inference via particle marginal Metropolis-Hastings and particle Gibbs samplers. Compared to the standard Gibbs sampler, the particle Gibbs sampler exhibits high precision per computational effort, which, combined with ease of implementation, makes it an attractive alternative to the standard Gibbs sampler. The technique is tested and demonstrated on real data (among others the novel electronic currency Bitcoin) with satisfactory results.

Oleg Lukashenko
Karelian Research center RAS

On convergence of maximum workload for some Gaussian queues

We consider the queueing system with constant service rate fed by an input, whose Gaussian component has stationary increments and variance belonging to a class of *regularly varying at infinity functions* with index $V \in (0, 2)$. The maximum of the workload process $Q(t)$ over a finite interval $[0,\ t]$ is an important performance measure of queueing systems. We present an asymptotic of $Q(t)$ as $t \to \infty$. More precisely, we prove that under appropriate scaling the maximum workload converges in probability to an explicitly given constant.

APPROXIMATION COMPLEXITY OF GAUSSIAN TENSOR PRODUCT-TYPE RANDOM FIELDS

A. A. KHARTOV

Consider a sequence of Gaussian tensor product-type random fields, given by

$$X_d(t) = \sum_{k \in \widetilde{\mathbb{N}}^d} \prod_{l=1}^{d} \lambda_{k_l}^{1/2} \xi_k \prod_{l=1}^{d} \psi_{k_l}(t_l), \quad t \in [0,1]^d, \quad d \in \mathbb{N},$$

where $(\lambda_i)_{i\in\widetilde{\mathbb{N}}}$ and $(\psi_i)_{i\in\widetilde{\mathbb{N}}}$ are all positive eigenvalues and eigenfunctions of covariance operator of process X_1, $(\xi_k)_{k\in\widetilde{\mathbb{N}}}$ are standard Gaussian random variables, and $\widetilde{\mathbb{N}}$ is a subset of natural numbers. For any $d \in \mathbb{N}$ sample paths of X_d almost surely belong to $L_2([0,1]^d)$ supplied with a norm $\|\cdot\|_{2,d}$.

The tuples $(\lambda_k, \psi_k) := \left(\prod_{l=1}^d \lambda_{k_l}, \prod_{l=1}^d \psi_{k_l}\right)$, $k \in \widetilde{\mathbb{N}}^d$, are eigenpairs of the covariance operator of X_d. We approximate X_d, $d \in \mathbb{N}$ with finite sum $X_d^{(n)}$ corresponding to n maximal eigenvalues λ_k, $k \in \widetilde{\mathbb{N}}^d$.

We investigate the logarithmic asymptotics of *average approximation complexity*

$$n_d^{avg}(\varepsilon) := \min\Big\{ n \in \mathbb{N} : \mathbb{E}\, \|X_d - X_d^{(n)}\|_{2,d}^2 \leqslant \varepsilon^2\, \mathbb{E}\, \|X_d\|_{2,d}^2 \Big\}$$

and *probabilistic approximation complexity*

$$n_d^{pr}(\varepsilon, \delta) := \min\Big\{ n \in \mathbb{N} : \mathbb{P}\left(\|X_d - X_d^{(n)}\|_{2,d}^2 > \varepsilon^2\, \mathbb{E}\, \|X_d\|_{2,d}^2 \right) \leqslant \delta \Big\},$$

when the *parametric dimension* $d \to \infty$, the *error threshold* $\varepsilon \in (0,1)$ is fixed, and the *confidence level* $\delta = \delta_{d,\varepsilon}$ may go to zero. Complementing recent results of M. A. Lifshits and E. V. Tulyakova (see [1]) we consider unexplored case, when the sequence $(\lambda_i)_{i\in\widetilde{\mathbb{N}}}$ is regularly and slowly decreasing to zero.

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St.Petersburg State University, Department of Mathematics and Mechanics.
E-mail address: AlexeyKhartov@gmail.com

The author is supported by the Program “Leading Scientific Schools” (project 1216.2012.1).

Generalized Gaussian Bridges

Tommi Sottinen, Adil Yazigi
University of Vaasa

Abstract:

A generalized bridge is the law of a stochastic process that is conditioned on linear functionals of its path. We consider two types of representations of such bridges: orthogonal and canonical. In the canonical representation the filtrations generated by the bridge process and the original process coincide. In the orthogonal representation the bridge is constructed from the entire path of the underlying process. The orthogonal representation is given for any continuous Gaussian process but the canonical representation is given only for so-called prediction-invertible Gaussian processes. Finally, we apply the canonical bridge representation to insider trading by interpreting the bridge from an initial enlargement of filtration point of view.

GOODNESS-OF-FIT TESTS FOR THE POWER FUNCTION DISTRIBUTION BASED ON PROPERTIES OF ORDER STATISTICS

KSENIA VOLKOVA

We construct integral and supremum type goodness-of-fit tests for the family of power distribution functions. Test statistics are functionals of $U-$empirical processes and are based on the classical characterization of power function distribution family belonging to Puri and Rubin. We describe the logarithmic large deviation asymptotics of test statistics under null-hypothesis, and calculate their local Bahadur efficiency under common parametric alternatives. Conditions of local optimality of new statistics are given.

St.Petersburg State University, Department of Mathematics and Mechanics
E-mail address: `efrksenia@yandex.ru`

Poissonization of Rice distribution by Data Augmentation in diffusion tensor imaging

DARIO GASBARRA *, JIA LIU † AND JUHA RAILAVO ‡

Abstract

Bayesian inference quantifies all parameters of interest or we say uncertainties by their probabilities. In this work a Bayesian hierarchical model is built to solve a specific problem in diffusion tensor imaging (DTI) of human brain. Using data augmentation and Markov chain Monte Carlo (McMC) methods, we successfully measure diffusion tensors in the whole region of signal amplitudes. Regularization method is introduced based on rotated-invariant Gaussian priors in configured random fields of the brain. After regularizations, the image noise including artifacts of tensor fields has reduced by measuring point- and block- information of the neighbors in the underlying mesh-network. We share a strong desire to find accurate signs in this work which can identify diffusion tensor images of human brain.

Key words: Rice noise, Poissonization, Data Augmentation, Gaussian random fields, regularization.

*Department of Mathematics and Statistics, University of Helsinki P.O. Box 68 FI-00014 Finland e-mail: dario.gasbarra@helsinki.fi

†Department of Mathematics and Statistics, University of Jyväskylä, P.O.Box (MaD) FI-40014 Finland e-mail: jia.liu@jyu.fi

‡HUS e-mail: juha.railavo@elisanet.fi

A simple time-consistent model for the forward density process

Johan Nykvist
KTH

Abstract:
In this talk a simple model for the evolution of the forward density of the future value of an asset is proposed. The model allows for a straightforward initial calibration to option prices and has dynamics that are consistent with empirical findings from option price data. The model is constructed with the aim of being both simple and realistic, and avoid the need for frequent re-calibration. The model prices of n options and a forward contract are expressed as time-varying functions of an (n+1)-dimensional Brownian motion and it is investigated how the Brownian trajectory can be determined from the trajectories of the price processes.

An approach based on particle filtering is presented for determining the location of the driving Brownian motion from option prices observed in discrete time. A simulation study and an empirical study of call options on the S&P 500 index illustrates that the model provides a good fit to option price data.

Inspection paradox: an application to loss and optical queues

L.Potakhina
(Joint work with K. De Turk and E.Morozov)

The renewal time (or inspection) paradox is well-known and can be found in many works. Roughly, it means that the mean stationary forward renewal time contains the 2nd moment of the renewal time and, in particular, may be bigger than the mean standard interrenewal time. In the work, we use numerical simulation to study the rate of convergence in the paradox for various renewal time distributions. A related form of renewal time paradox concerns a renewal time covering an increasing instant t. In the limit, this quantity equals double mean stationary remaining time. As we show this problem is of interest in practice.

First we apply the renewal time paradox to analyze the loss probability a class of non-conventional loss systems. In such a system, each customer has both service time and a size, and the system has (potentially) infinite capacity for the queue-size but a finite capacity M for the total size of the awaiting customers. Thus, the arriving customer is lost if he meets total size N in the system and his size v is such that $v + N \geq M$. By the numerical simulation we confirms a conjecture that for M large the mean lost size approaches to the value obtained from the renewal time paradox.

Then we consider system with the optical buffers. In the system, signals travel from host to host in the form of light and buffering by means of a set of fiber delay lines (FDL). So the set of possible waiting times is not a continuum (like in a classic queueing system), but a denumerable set, where each value corresponding to the length of a delay line. As a result, in general arriving signals have to wait for service longer than in a classic case, since their waiting times has to be in that denumerable set. A sufficient stability condition for the systems with optical buffers has been recently obtained. We verify by simulation a tighter sufficient condition which stems from the renewal time paradox.

Markov chain Monte Carlo for computing probabilities of rare events in a heavy-tailed random walk

Thorbjörn Gudmundsson
KTH

Abstract:
An algorithm based on Markov chain Monte Carlo (MCMC) is proposed to compute probabilities of rare events of a heavy-tailed random walk. The algorithm is based on sampling from the conditional distribution given the rare event using MCMC and extracting the normalizing constant. The algorithm is proved to be strongly efficient for computing tail probabilities of a random walk with heavy-tailed distributed steps.

DUOPOLY IN QUEUEING SYSTEMS

A. Mazalova

A non-cooperative two-person game which is related to the queueing system $M/M/2$ is considered. There are two services which serve the stream of customers with exponential distribution with parameters μ_1 and μ_2 respectively. The stream forms the Poisson process with intensity λ. Players declare prices for service c_1 and c_2 respectively, and visitors choose the service with minimal costs. The cost consists of the price for the service and the cost for the time in the queue. So, the incoming flow divides on two Poisson flows with the intensities λ_1 and λ_2, where $\lambda_1 + \lambda_2 = \lambda$. The problem of pricing and determining the optimal intensity for each firm in the competition and cooperation is solved. The existence of the only Nash equilibrium is shown and the equations for the strategies that make this equilibrium are derived. The cooperative approach is considered for the case of two or three players. The value of the characteristic function for the individual player is taken equal to the payoff of the player in Nash equilibrium. Also the increase of the number of players is carried out.

*Key words:*Duopoly, equilibrium prices, queueing system.

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On critical Gaussian multiplicative chaos

Miika Nikula
University of Helsinki

Abstract:
Gaussian multiplicative chaos is a natural construction of a random multifractal measure. The construction depends essentially on a single parameter $\beta > 0$ that controls the dimension of the support of the measure. It is well known that for $\beta^2 \geq 2$ the classical construction degenerates and resulting measure is almost surely null. However, the distributional scaling relations satisfied by multiplicative chaos measures still make sense even in the degenerate case, and thus there has been a search for a construction which would result in random measures satisfying the appropriate scaling relations in the case $\beta^2 \geq 2$. Very recently, such a construction was given in the critical case $\beta^2 = 2$. In this talk we will study some geometric properties of this critical Gaussian multiplicative chaos measure. Especially, we will show that there exists a set of full measure that has Hausdorff dimension 0 (with respect to the standard metric), and give a bound for the modulus of continuity of the cumulative distribution function of the measure.

Large repeated games with incomplete information: asymptotics of the value.

Sandomirskii Fedor

Repeated zero-sum games with incomplete information (RGII) were introduced by R. Aumann and M. Maschler in the sixties (see [1] and [3] for details). Their aim was to develop a theory of multistage two-player interactions with participants having different amount of information about the interaction. Multistage structure allows players to obtain additional information about the interaction observing their opponents actions at previous stages. Zero-sum property means that players have completely opposite goals.

Let us describe such a game $G_N(p)$. Before the game starts a random state θ is chosen from the set of states Θ according to a probability distribution p on Θ. Player 1 is informed of θ and Player 2 knows only p. Further at each stage $n = 1, ...N$ Player 1 and Player 2 simultaneously select their actions i_n and j_n from the sets of actions I and J using the information they have at this stage. Before the next stage the selected actions are publicly announced. One-stage payoff (i.e., the contribution of this stage to the total gain of Player 1 and to the total loss of Player 2) is given by $A_\theta(i_n, j_n)$, where A_s is the one-stage payoff function at a state s. After the last stage Player 1 receives the sum $\sum_{n=1}^{N} A_\theta(i_n, j_n)$ from Player 2. Note that Player 2 does not know stage payoffs during the game.

It is natural for players to use randomized actions (for example, this allows Player 1, who does not want Player 2 to guess θ, to reveal his private information gradually). Hence a (behavioral) strategy of Player 1 is a collection $\sigma = \{\sigma_n^s\}_{n=1,..N,\ s\in\Theta}$. Here σ_n^s is a stochastic kernel which maps a history $h_n = (i_m, j_m)_{m=1}^{n-1}$ to a probability measure on I. This measure is the conditional distribution of i_n given a history and $\theta = s$. A strategy of Player 2 is $\tau = \{\tau_n\}_{n=1,..N}$. Strategies and p generate the probability measure $\mathbb{P}_{(p,\sigma,\tau)}$ on $\Theta \times (I \times J)^N$. The average total gain of Player 1 is given by $g(p, \sigma, \tau) = \mathbb{E}_{(p,\sigma,\tau)} \sum_{n=1}^{N} A_\theta(i_n, j_n)$. The average total gain if both players are doing the best is called the value of the game, i.e.,

$$\text{val}[G_N(\mu)] = \sup_\sigma \inf_\tau g(p, \sigma, \tau) = \inf_\tau \sup_\sigma g(p, \sigma, \tau).$$

Here the second equality is due to famous min-max theorem (holds under some additional assumptions like finiteness of Θ, I and J).

Assume that the non-revealing game G_N^{NR} where Player 1 forgets the state (i.e., now σ_n^s does not depend on s for any n) has zero value for any p (denote this hypothesis by $(\star)$). It means that the only strategic advantage of Player 1 in $G_N(p)$ is his information. Therefore under this assumption the value can be viewed as a price of information. One of the main questions in theory of RGII is about asymptotic behavior of the value as $N \to \infty$. It is well-known that if Θ, I and J are finite and $(\star)$ holds, then the value can not grow faster then $CN^{\frac{1}{2}}$ and this upper bound is exact, i.e., there are games with such asymptotic behavior of the value (see [1] and [3]). This is so-called "$\sqrt{N}$-law" established by R. Aumann and M. Maschler.

Our aim is to examine the case of countable Θ. Let $\Gamma(\Theta)$ denote the set of all RGII such that: Θ is the set of states; payoff-functions are uniformly bounded; hypothesis $(\star)$ holds (we need some additional technical assumptions implying existence of the value too). For any probability distribution p over Θ define the following measure of uncertainty

$$Z_\delta(p) = \sum_{s\in\Theta} p(\{s\}) \left[\ln\left(\frac{1}{p(\{s\})}\right)\right]^{\frac{1}{2}-\delta}, \quad \delta \leq \frac{1}{2}.$$

Note that $\delta = -\frac{1}{2}$ corresponds to the Shannon entropy.

Theorem: *For any probability distribution p over countable set Θ the following relation holds*

$$\sup_{G\in\Gamma(\Theta)} \limsup_{N\to\infty} \frac{\ln \mathrm{val}[G_N(p)]}{N} = \frac{1}{2} + \inf\left\{\delta \in [0,1/2] \mid Z_\delta(p) < \infty\right\}.$$

Therefore for a distribution p with heavy tails there are games with the value growing faster then $N^{\frac{1}{2}}$. The proof is based on the analysis of the variation of measure-valued martingales arising as conditional distributions of θ given a history $h_n = (i_m, j_m)_{m=1}^{n-1}$ (the main ideas of this approach are from [2] and [3]).

It is provided that the "anomalous growth" is possible only for large games. Using approximation ideas we show that if Θ is arbitrary but I and J are finite, then the following upper bound holds

$$\mathrm{val}[G_N(p)] < C\sqrt{N \ln N}$$

with some C independent of N and p.

The study is supported by the grant 10-06-00368-a of Russian Foundation for Basic Research and by the Chebyshev Laboratory (Faculty of Mathematics and Mechanics, St Petersburg State University) under the grant 11.G34.31.0026 of the Government of the Russian Federation.

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Optimal strategies in two-sided best-choice game with age preferences

A.A. Ivashko

Institute of Applied Mathematical Research
Karelian Research Center of RAS
185910, Russia, Petrozavodsk, Pushkinskaya str., 11
http://mathem.krc.karelia.ru
aivashko@krc.karelia.ru

Keywords: mutual mate choice, equilibrium, threshold strategy

Abstract

The two-sided mate choice model of Alpern, Katrantzi and Ramsey (2010) [3] is considered. In the problem individuals from two distinct groups (for example, males and females) want to form a long-term relationship with a member of the other group, i.e. to form a couple. Each group has steady state distribution for the age of individuals. In the model males and females have lifetime m and n respectively. It is assumed that the total number of unmated males is greater than the total number of unmated females and $m \geq n$. The discrete time game is considered. In the game unmated individuals from different groups randomly meet each other in each period. If they accept each other, they form a couple and leave the game, otherwise they go into the next period unmated and older. It is assumed that individuals of both sexes enter the game at age 1 and stay until they are mated or males (females) pass the age m (n). The payoff of mated player is the number of future joint periods with selected partner. The aim of each player is to maximize the expected payoff. Other two-sided mate choice models were considered in papers [1, 2, 4–7]. Alpern, Katrantzi and Ramsey (2010) [3] derive properties of equilibrium threshold strategies and analyse the model for small m and n. In this paper we derive analytically the equilibrium threshold strategies and investigate players' payoffs for the case $n = 3$ and large m.

The work is supported by Russian Fund for Basic Research (project 13-01-91158-ГФЕН_a) and the Division of Mathematical Sciences of RAS (the program "Algebraic and combinatorial methods of mathematical cybernetics and new information system").

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An introduction to disability insurance modelling

Björn Löfdahl
KTH

Abstract:
A health and disability insurance policy entitles to a monthly benefit as compensation for a reduction or loss of income due to sickness or accident. In order to calculate the premium (price) of such policies, the insurer needs estimates of the future rates of disability inception and recovery. This talk gives an introduction to disability insurance and presents a framework for modelling inception and recovery that allows for concrete interpretations.